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Research Article

Electromagnetic wave propagation in a rectangular waveguide filled with a double-negative (DNG) metamaterial

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Abstract

This article presents an analysis of electromagnetic wave propagation in a model of rectangular waveguide when filled with a double-negative metamaterial. The field components and dispersion relation of the electromagnetic waves are calculated, analytically. The dispersion relation is numerically analysed by plotting the normalized squared phase velocity against the propagation frequency for the variation of parameters of waveguide and the wave. The dominant mode is found to be TE^{10} mode. The dispersion curves show propagation and non-propagation characteristics for larger dimensions and for dominant mode. Some aspects were found to be much different from a conventional rectangular wave-guide.

Keywords:
Rectangular waveguide, double-negative metamaterial, wave propagation.

1. Introduction

Rectangular waveguide loaded with various modern media has been the topic of current interest for the microwave communication device applications such as filters, isolators, couplers, sensors etc. [1–4]. In modern microwave materials, the double-negative (DNG) metamaterials show very interesting properties as compared to the conventional double-positive (DPS) dielectrics. Vesselago [5] suggested the plane wave propagation in a medium containing simultaneously negative permittivity and permeability. This idea then experimentally realized by Pendry et al. [6–8], Smith et al. [9], and Shelby et al. [10]. Now, the DNG materials have been studied in different waveguides and cavities for different applications such as low-pass, high-pass and multiband filters etc. [11–15]. These applications have been revealed by various authors. For example, Weng et al. [1] used accurate rigorous modal theory to study rectangular waveguide filled with multilayer right and left handed metamaterials and shown the multiband propagation properties below cut-off. Moradi [2] studied a rectangular waveguide filled with anisotropic medium by using electrostatic theory of the wave propagation and proposed its applications in microwave regime. In another work, Weng et al. [3] investigated the rectangular waveguide filled with graded inhomogeneous metamaterial for the wave propagation and suggested a new approach for the waveguide miniaturization.

In above discussed waveguides, the medium filled in the waveguide played the key role for their applications. Therefore, in the current proposed study, we analyse a rectangular waveguide filled with a linear, homogeneous and isotropic double-negative (DNG) metamaterial for the possible propagation properties and applications. This article is organized as follows: section (1) represents the introduction to the current study; section (2) includes the theoretical analysis of the proposed waveguide based on electromagnetic wave theory; section (3) aims to present the numerical results with discussion, and section (4) focuses on the conclusion.

2. Theoretical analysis

Consider a rectangular waveguide with perfect electrical conducting (PEC) walls as shown in figure (1). The planar symmetry of the waveguide is extended in xz-plane with dimensions $y = a$ and $x = b$, whereas, the waveguide is extended infinitely along z-direction. An electromagnetic wave is considered to propagate in the z-direction with the plane wave solution profile $e^{i(\omega t-kz)}$. The medium inside the boundary walls of the waveguide is a linear, homogeneous and isotropic double-negative (DNG) material.

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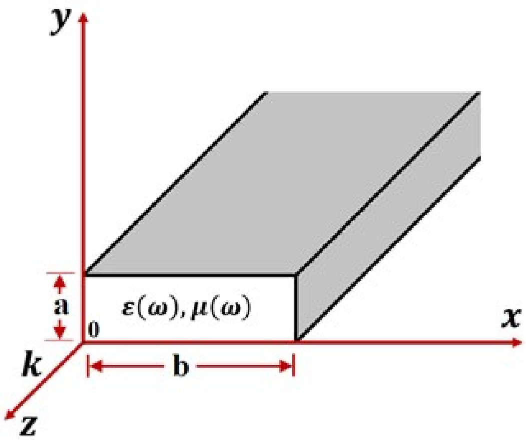

Figure 1: Model of a rectangular waveguide filled with DNG metamaterial.

A left-handed or DNG material is so-called frequency dependant material, characterized by the following expressions of relative permittivity and relative permeability [11–14].

$$\varepsilon_D(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

$$\mu_D(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_r^2} \tag{2}$$

where ω is the frequency of propagation, ω_p is the plasma frequency of long metallic wires (LMW), ω_r is the resonance frequency of the split ring resonators (SRRs) and F is the filling fraction of DNG material structure. Maxwell equations for the DNG material can be written as

$$\nabla \times H = i\omega\varepsilon_\circ\varepsilon_D(\omega)E \tag{3}$$

$$\nabla \times E = -i\omega\mu_\circ\mu_D(\omega)H \tag{4}$$

Coupling the above Maxwell equations, we get the following wave equations for E and H

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = k_D^2 E \tag{5}$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = k_D^2 H \tag{6}$$

Where $k^2 - \omega\varepsilon_D(\omega)\mu_D(\omega) = k_D^2$ Writing equations (3) and (4) in component form, the following sets of equations are obtained

$$\begin{cases} \dfrac{\partial H_z}{\partial y} + ikH_y = i\omega\varepsilon_D(\omega)E_x & (7) \\ -\dfrac{\partial H_z}{\partial x} - ikH_x = i\omega\varepsilon_D(\omega)E_y & (8) \\ \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = i\omega\varepsilon_D(\omega)E_z & (9) \end{cases}$$

$$\begin{cases} \dfrac{\partial E_z}{\partial y} + ikE_y = -i\omega\mu_D(\omega)H_x & (10) \\ -\dfrac{\partial E_z}{\partial x} - ikE_x = -i\omega\mu_D(\omega)H_y & (11) \\ \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -i\omega\mu_D(\omega)H_z & (12) \end{cases}$$

Solving the above set of equations for the field components H_x, E_x, H_y, E_y in terms of H_z and E_z, we get the following expressions

$$H_x = \frac{i}{k_D^2}\left(k\frac{\partial H_z}{\partial x} + \omega\varepsilon_D(\omega)\frac{\partial E_z}{\partial y}\right) \tag{13}$$

$$E_x = \frac{i}{k_D^2}\left(\omega\mu_D(\omega)\frac{\partial H_z}{\partial y} + k\frac{\partial E_z}{\partial x}\right) \tag{14}$$

$$H_y = \frac{i}{k_D^2}\left(k\frac{\partial H_z}{\partial y} + \omega\varepsilon_D(\omega)\frac{\partial E_z}{\partial x}\right) \tag{15}$$

$$E_y = \frac{i}{k_D^2}\left(\omega\mu_D(\omega)\frac{\partial H_z}{\partial x} - k\frac{\partial E_z}{\partial y}\right) \tag{16}$$

The solution of equation (5) is obtained for E_z by using separation of variables method, given by

$$E_z = (c_1 \cos Ax + c_2 \sin Ax)(c_3 \cos By + ic_4 \sin By) \tag{17}$$

where $A^2 = -k_D^2 - B^2 and c_1, c_2, c_3$and c_4 are constants which can be evaluated by using appropriate boundary conditions. Now, using the boundary conditions, (i.e. the vanishing of the tangential field component at the surface of a PEC), in equation (17), we get the following solution

$$E_z = E_o \sin\left(\frac{n\pi x}{b}\right)\sin\left(\frac{m\pi y}{a}\right) \tag{18}$$

where $n = 0, 1, 2, \ldots$ and $m = 0, 1, 2 \ldots$, $A = n\pi/b$, $B = m\pi/a$ and $E_\circ$ is the amplitude of E_z.

2.1. TM modes

There is no magnetic field component in the direction of propagation of wave for a transverse magnetic (TM) mode, therefore, setting $H_z = 0$ in equations (13)-(16), we get the following field vectors for the TM wave modes

$$H_x = \frac{i\omega}{k_D^2}\varepsilon_D(\omega)\frac{\partial E_z}{\partial y} \tag{19}$$

$$E_x = \frac{ik}{k_D^2}\frac{\partial E_z}{\partial x} \tag{20}$$

$$H_y = \frac{i\omega}{k_D^2}\varepsilon_D(\omega)\frac{\partial E_z}{\partial x} \tag{21}$$

$$E_y = -\frac{ik}{k_D^2}\frac{\partial E_z}{\partial y} \tag{22}$$

Using the solution (18) in above equations, we get the following field components for the TM wave modes

$$H_y = \frac{i\omega\varepsilon_D(\omega)}{k_D^2}\frac{n\pi}{b}\left\{E_o\cos\left(\frac{n\pi x}{b}\right)\sin\left(\frac{m\pi y}{a}\right)\right\} \quad (23)$$

$$E_y = \frac{ik}{k_D^2}\frac{m\pi}{a}\left\{E_o\sin\left(\frac{n\pi x}{b}\right)\cos\left(\frac{m\pi y}{a}\right)\right\} \quad (24)$$

$$E_x = \frac{ik}{k_D^2}\frac{n\pi}{b}\left\{E_o\cos\left(\frac{n\pi x}{b}\right)\sin\left(\frac{m\pi y}{a}\right)\right\} \quad (25)$$

$$H_x = \frac{-i\omega\varepsilon_D(\omega)}{k_D^2}\frac{m\pi}{a}\left\{E_o\sin\left(\frac{n\pi x}{b}\right)\cos\left(\frac{m\pi y}{a}\right)\right\} \quad (26)$$

To find the expression of the dispersion relation for the TM modes, we use $A = n\pi/b$, $B = m\pi/a$ in $A^2 = -k_D^2 - B^2$ as mentioned in equations (17) and (18), we get

$$k^2 = \frac{\omega^2}{c^2}\varepsilon_D(\omega)\mu_D(\omega) - \left(\frac{n\pi}{b}\right)^2 - \left(\frac{m\pi}{a}\right)^2 \quad (27)$$

2.2. TE Modes

For the transverse electric (TE) Modes, there is no electric field component in the direction of propagation of wave. Therefore, we use $E_z = 0$ in equations (13)-(16), we get the following field vectors for the TE wave modes

$$H_x = \frac{ik}{k_D^2}\frac{\partial H_z}{\partial x} \quad (28)$$

$$E_x = \frac{i\omega}{k_D^2}\mu_D(\omega)\frac{\partial H_z}{\partial y} \quad (29)$$

$$H_y = \frac{ik}{k_D^2}\frac{\partial H_z}{\partial y} \quad (30)$$

$$E_y = \frac{i\omega}{k_D^2}\mu_D(\omega)\frac{\partial H_z}{\partial x} \quad (31)$$

For the TE modes, we solved equation (6) for H_z by using separation of variables method and use the same mathematical procedure as described in section (2.1) to find the following solution

$$H_z = H_o\cos\left(\frac{n\pi x}{b}\right)\cos\left(\frac{m\pi y}{a}\right) \quad (32)$$

where $H_\circ$ is the amplitude of H_z. The corresponding field components for the TE modes are given by

$$E_x = -\frac{i\omega\mu}{k_D^2}\left(\frac{m\pi}{a}\right)\left\{H_o\sin\left(\frac{n\pi x}{b}\right)\sin\left(\frac{m\pi y}{a}\right)\right\} \quad (33)$$

$$E_y = -\frac{i\omega\mu}{k_D^2}\left(\frac{m\pi}{a}\right)\left\{H_o\cos\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi x}{b}\right)\right\} \quad (34)$$

$$H_x = -\frac{ik}{k_D^2}\left(\frac{m\pi x}{b}\right)\left\{H_o\cos\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi x}{b}\right)\right\} \quad (35)$$

$$H_y = -\frac{ik}{k_D^2}\left(\frac{n\pi}{a}\right)\left\{H_o\sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi x}{b}\right)\right\} \quad (36)$$

In this case, the dispersion relation is the same as that of the TM wave mode, given by

$$k^2 = \frac{\omega^2}{c^2}\varepsilon_D(\omega)\mu_D(\omega) - \left(\frac{n\pi}{b}\right)^2 - \left(\frac{m\pi}{a}\right)^2 \quad (37)$$

3. Results and discussion

To analyse the proposed rectangular waveguide for the wave propagation characteristics, the dispersion relation given in equation (37) is mathematically manipulated for the normalized squared phase velocity, given by

$$\frac{v_\varphi^2}{c^2} = \left\{\varepsilon_D(\omega)\mu_D(\omega) - \left(\frac{cn\pi}{\omega b}\right)^2 - \left(\frac{cm\pi}{\omega a}\right)^2\right\}^{-1} \quad (38)$$

To investigate the dispersion characteristics of the electromagnetic wave, it is more convenient to plot normalized squared phase velocity against frequency, instead of usual wave vector-frequency plot [11]. First, we find the frequency range for which both permittivity and permeability of the DNG metamaterial have simultaneously negative values. For this purpose, a graph of equations (1) and (2) are plotted for the permittivity and permeability against the frequency, as shown in figure (2), for the parameters $F = 0.56, \omega_r = 4 \times 10^9 Hz and \omega_p = 10 \times 10^9 Hz$ [11–15]. The frequency range is found to be $4 \times 10^9 Hz$ and $6 \times 10^9 Hz$ which is in microwave frequency range.

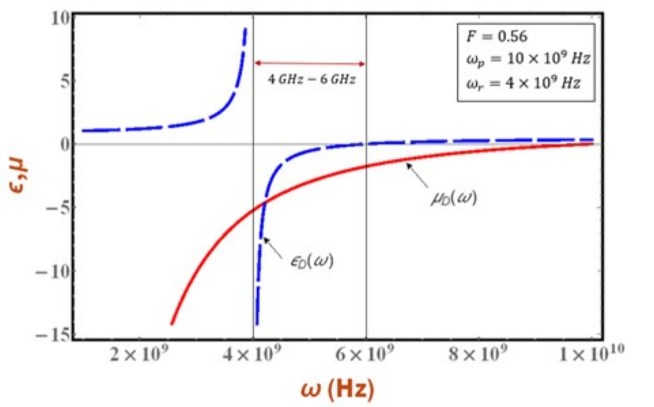

Figure 2: A graph between relative permittivity/permeability and frequency to find the existence range of a DNG material.

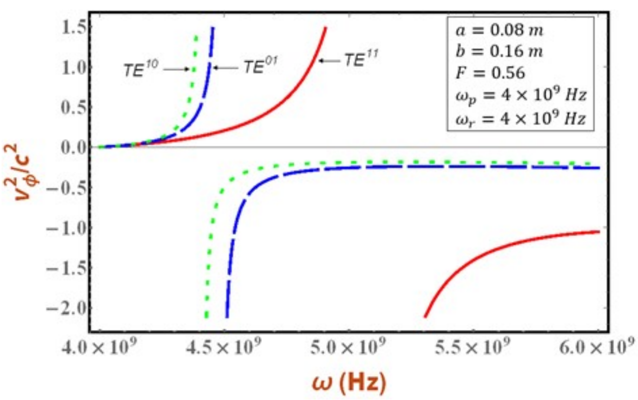

Figure 3: A plot of $\frac{v_\varphi^2}{c^2}$ versus ω for various waveguide modes and for the rectangular waveguide filled with DNG material

The dispersion characteristics are obtained by plotting equation (38) for $\frac{v_\varphi^2}{c^2}$ versus ω within the existence range of the DNG metamaterial, as shown in figure (3). This figure shows the dispersion curves for various TE^{mn} modes i.e. TE^{10}, TE^{01}, and TE^{11} modes. Each curve shows two branches separated by a cut-off. The branch of each curve with $\frac{v_\varphi^2}{c^2} > 0$ shows the propagation of electromagnetic waves through waveguide, whereas the branch of each curve with $\frac{v_\varphi^2}{c^2} < 0$ represents non-propagation region. The frequency separating the propagation and non-propagation regions is the cut-off frequency, given by

$$\omega_c = \left[\frac{1}{\varepsilon_D(\omega)\mu_D(\omega)}\left\{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right\}\right]^{1/2} \quad (39)$$

Within $4GHz - 6GHz$, the wave propagation is observed for $\omega < \omega_c$, therefore, the waveguide is behaving like a low-pass filter which is different from a conventional rectangular waveguide (a waveguide filled with a usual right-handed dielectric material) for which there is no propagation found in the frequency range under consideration (see figure (4)). This property of the proposed structure may be useful as filter, sensor etc., within the microwave frequency band. It is clear from the graph that the cut-off for various wave modes are different and TE^{10} is the dominant mode.

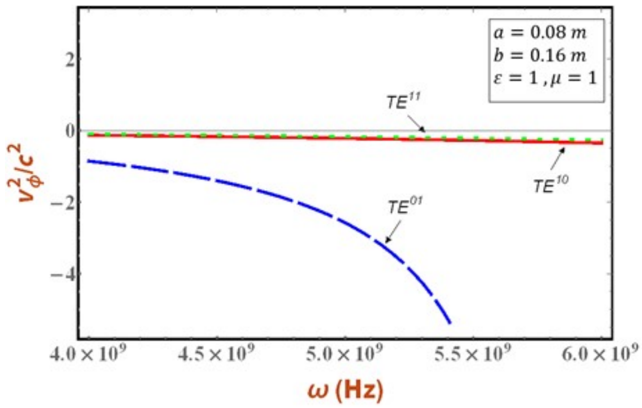

Figure 4: A plot of $\frac{v_\varphi^2}{c^2}$ versus ω for various waveguide modes and for the rectangular waveguide filled with DPS material.

To discuss the effect of the dimensions of waveguide on propagation characteristics, the dispersion relation (38) is again plotted for $\frac{v_\varphi^2}{c^2}$ versus ω for different values of the dimensions a and b and for dominant mode TE^{10}, as shown in figure (5). The dispersion curves show the same behaviour as discussed in figure (3). It is seen that by increasing the dimensions of the waveguide, the cut-off shifts to a higher frequency value and the propagation region of frequency also increases. It can further be concluded that the cut-off is a function of both, dominant mode and the waveguide dimensions. Such properties of a waveguide are useful as filters, sensor and maybe helpful in the development of microwave radars and antennas technology.

4. Conclusion

In this work, a rectangular waveguide loaded with a double-negative (DNG) metamaterial is proposed to study the propagation characteristics of the electromagnetic waves. In this connection, the dominant mode is found to be TE^{10} but the wave propagation characteristics are much different from a conventional waveguide filled with right handed dielectric. Within the existence range of a DNG material, the waveguide behaves as a low pass filter with a cut-off that increases by increasing the dimensions of the waveguide. Whereas, conventional waveguide does not show any propagation for the choice of same dimensions and the frequency band. Thus, for a particular choice of dimensions, the proposed waveguide maybe used in microwave applications such as filters, sensors, and isolators etc. within a frequency range where the conventional waveguide, loaded with right handed material, does not show any propagation.

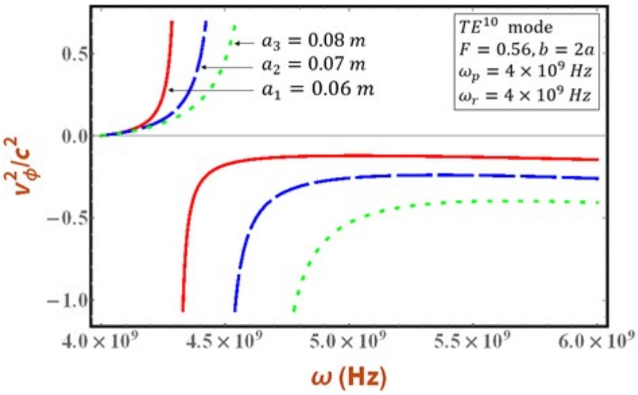

Figure 5: A plot of $\frac{v_\varphi^2}{c^2}$ versus ω for various waveguide dimensions and for the dominant mode TE^{10} of the rectangular waveguide filled with DNG material

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