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Research Article

TE and TM surface waves at the interface of a left-handed metamaterial

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Abstract

In this paper, we present an analytical treatment for the electromagnetic surface waves propagating at the interface of a linear right-handed material and a linear left-handed material. In this connection, we use Maxwell equations to find the electric and magnetic field components corresponding to the transverse electric (TE) and transverse magnetic (TM) wave polarizations. Electromagnetic boundary conditions are then employed on the field components at the interface to find the dispersion relations for both TE and TM waves. These dispersion relations are reduced to the case of TE and TM surface waves propagation along an interface between two different right-handed media. The dispersion relations are then analyzed numerically by plotting them for the propagation frequency and the wave vector for both wave modes. Some features of waves propagating at these interfaces are found to be useful in microwave device applications.

Keywords:
TE surface waves, TM surface waves, left-handed material.

1. Introduction

The fabrication and investigation of novel materials developed modern applied physics. Among these materials, the best known examples are the metamaterials, photonic band-gap materials and birefringent media, due to their unique electromagnetic properties [1–8]. The class of metamaterials with simultaneously negative permittivity and permeability in a specific range of frequency show some peculiar electrodynamic properties [4, 9, 10] and therefore used to improve the working of optical and microwave devices etc. These metamaterials are known as left-handed materials (LHMs) [11] because the wave vector makes a left-handed set of vectors with electric and magnetic fields. In 1968, Vesselago [12] first time gave the idea of left-handed materials and suggested some unusual phenomena including negative refraction. Later, due to the work of Pendry et al. [13–15], Smith et al. [9] and Shelby et al. [4], these materials were physically realized [9] by fabricating artificially engineered structures showing the simultaneously negative permittivity and permeability in a certain range of frequency. Since then, various authors reported a lot of research work in connection with these left-handed materials for their practical applications to optical and microwave devices (e.g., [5, 16, 17])

In the recent years, a considerable work has been done to study the surface and guided waves on interfaces of various modern materials [18–20] and employed on waveguide and sandwich structures for their development and applications to the communication devices etc. For example, Shadrivov et al. [11] study the surface at the interface of nonlinear left-handed media. In this analysis, they discussed the possibility of tuning the group velocity of the wave with the help of nonlinearity of the LHM. Maimistov and Lyashko [18] theoretically studied the surface waves propagating along an interface between an isotropic dielectric and a topological insulator. They derived the dispersion relation for both TE and TM modes and found that the two modes are in superimposed form. El-Khozondar et al.[21] investigated the guiding electromagnetic waves along ferroelectric / metamaterial interface. This work is related to the derivation and numerical solution of the dispersion relation, which shows the dependence of dispersion characteristics on nonlinearity of ferroelectric material.

In the present work, we theoretically analysed both TE and TM surface waves propagating along an interface between a linear right-handed material (RHM) and a linear left-handed material (LHM). A planar symmetry is used in the theoretical model to describe TE and TM wave modes. The dispersion relation for RHM/LHM interface is derived and is reduced to RHM/RHM interface. This work may find its importance for the design and

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development of different communication devices, such as integrated circuits, transmission lines and antennas systems etc., operating at microwave frequencies.

2. Theoretical analysis

Surface electromagnetic waves at the interface between a right handed and a left-handed medium, as shown in Figure 1 had been studied

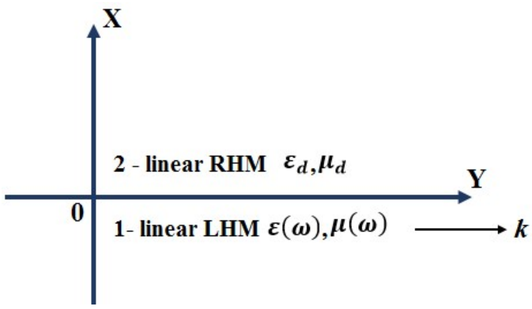

Figure 1: Surface waves at RHM/LHM interface

An electromagnetic surface wave is propagating in y- direction with propagation constant k and frequency ω at conventional linear RHM (at $x < 0$) and a LHM (at $x > 0$). Both the media are infinitely extended in yz-plane. The RHM is specified by the constant (positive) relative permittivity and permeability ε_d and μ_d, whereas, for LHM, the negative values of relative permittivity and permeability are given by the frequency dependent functions given by

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{1}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_r^2} \tag{2}$$

where ω_p is the plasma frequency, ω_r is the resonance frequency and F is the filling factor [9]. Here, without loss of generality, the damping terms are not considered [19]. The field profiles for transvers electric (TE) and transverse magnetic (TM) waves have the following forms:

$$\text{TE Waves}: \begin{cases} E = [0, 0, E_z(\omega, x)]\, e^{i(\omega t - ky)} \\ H = \left[H_x(\omega, x), H_y(\omega, x), 0\right] e^{i(\omega t - ky)} \end{cases} \tag{3}$$

$$\text{TM Waves}: \begin{cases} E = \left[E_x(\omega, x), E_y(\omega, x), 0\right] e^{i(\omega t - ky)} \\ H = [0, 0, H_z(\omega, x)]\, e^{i(\omega t - ky)} \end{cases} \tag{4}$$

2.1. *Field components for left-handed material at $x < 0$*

To obtain the electromagnetic wave equation for the left-handed material, we first take the Maxwell field equations for the LHM as

$$\nabla \times H = i\omega\varepsilon_\circ\varepsilon(\omega)E \tag{5}$$

$$\nabla \times E = -i\omega\mu_\circ\mu(\omega)H \tag{6}$$

For the case of TE surface waves, equations 3 - 6 have been used to obtain the following wave equation for E_{Z_1}:

$$\frac{d^2E_{z_1}}{dx^2} - k^2E_{z_1} + k_\circ^2\mu(\omega)\varepsilon(\omega)E_{z_1} = 0 \tag{7}$$

where $K_\circ^2 = \frac{\omega^2}{c^2}$. The solution of the above wave equation is given by

$$E_{z_1} = a_1e^{k_1x} + a_2e^{-k_1x} \tag{8}$$

Here $K_1 = \left[K^2 - K_\circ^2\mu(\omega)\varepsilon(\omega)\right]^{\frac{1}{2}}$. a_1, a_2 are arbitrary constants and can be evaluated from the boundary conditions. For LHM (region 1), the following TE field components $E_{Z_1}(\omega, x)$, $H_{x_1}(\omega, x)$, and $H_{y_1}(\omega, x)$ are obtained from (5), (6) and (8), and here we assume that the field penetration length in LHM is much shorter than its dimensions:

$$E_{z_1}(\omega, x) = a_1e^{k_1x} \tag{9}$$

$$H_{x_1}(\omega, x) = \frac{k}{\omega\mu_\circ\mu(\omega)}(a_1e^{k_1x}) \tag{10}$$

$$H_{y_1}(\omega, x) = \frac{-ik_1}{\omega\mu_\circ\mu(\omega)}(a_1e^{k_1x}) \tag{11}$$

A similar mathematical treatment can be performed to calculate the following field components corresponding to TM waves:

$$H_{z_1}(\omega, x) = a_1'e^{k_1x} \tag{12}$$

$$E_{x_1}(\omega, x) = \frac{-k}{\omega\varepsilon_\circ\varepsilon(\omega)}(a_1'e^{k_1x}) \tag{13}$$

$$E_{y_1}(\omega, x) = \frac{ik_1}{\omega\varepsilon_\circ\varepsilon(\omega)}(a_1'e^{k_1x}) \tag{14}$$

2.2. *Field components for linear dielectric at $x > 0$*

To obtain the electromagnetic wave equation for the linear dielectric medium, we first take the Maxwell field equations for the LHM as

$$\nabla \times H = i\omega\varepsilon_\circ\varepsilon_dE \tag{15}$$

$$\nabla \times E = -i\omega\mu_\circ\mu_dH \tag{16}$$

For the case of TE surface waves, equations (15) and (16) have been used to obtain the following wave equation for E_{z_2} :

$$\frac{d^2E_{z_2}}{dx^2} - k^2E_{z_2} + k_\circ^2\varepsilon_d\mu_dE_{z_2} = 0 \tag{17}$$

The solution of the above wave equation is given by

$$E_{z_2} = c_1e^{k_2x} + c_2e^{-k_2x} \tag{18}$$

Here $K_2 = \left[K^2 - K_\circ^2 \varepsilon_d \mu_d\right]^{\frac{1}{2}}$. c_1 and c_2 are arbitrary constants and can be evaluated from the boundary conditions. For linear dielectric medium (region 2), the following TE field components $E_{z_2}(\omega, x)$, $H_{x_2}(\omega, x)$ and $H_{y_2}(\omega, x)$ are obtained from (15), (16) and (18), and here we assume that the field penetration length in linear dielectric is much shorter than its dimensions:

$$E_{Z_2}(\omega, x) = c_2 e^{-k_2 x} \tag{19}$$

$$H_{x_2}(\omega, x) = \frac{k}{\omega \mu_\circ \mu_d}(c_2 e^{-k_2 x}) \tag{20}$$

$$H_{y_2}(\omega, x) = \frac{ik_2}{\omega \mu_\circ \mu_d}(c_2 e^{-k_2 x}) \tag{21}$$

A similar mathematical treatment can be performed to calculate the following field components corresponding to TM waves:

$$H_{Z_2}(\omega, x) = c_2' e^{-k_2 x} \tag{22}$$

$$E_{x_2}(\omega, x) = \frac{-k}{\omega \varepsilon_\circ \varepsilon_d}(c_2' e^{-k_2 x}) \tag{23}$$

$$E_{y_2}(\omega, x) = \frac{-ik_2}{\omega \varepsilon_\circ \varepsilon_d}(c_2' e^{-k_2 x}) \tag{24}$$

2.3. The dispersion relation

To find the dispersion relation for the TE and TM waves, we employ the following boundary conditions for the continuity of the tangential field components at $x = 0$:

$$\text{TM Waves :} \begin{cases} E_{z_1}|_{x=0} &= E_{z_2}|_{x=0} \\ H_{y_1}|_{x=0} &= H_{y_2}|_{x=0} \end{cases} \tag{25}$$

$$\text{TM Waves :} \begin{cases} H_{z_1}|_{x=0} &= H_{z_2}|_{x=0} \\ E_{y_1}|_{x=0} &= E_{y_2}|_{x=0} \end{cases} \tag{26}$$

Using the values of $E_{z_1}(\omega, x)$, $E_{z_2}(\omega, x)$, $H_{y_1}(\omega, x)$ and $H_{y_2}(\omega, x)$ from equations (9), (11), (19) and (21) in equation (25) to obtain the following dispersion relation of the TE surface waves at the interface between a linear dielectric and a left-handed medium:

$$\frac{k_1}{\mu(\omega)} + \frac{k_2}{\mu_d} = 0 \tag{27}$$

Similarly, using the values of $H_{z_1}(\omega, x)$, $H_{z_2}(\omega, x)$, $E_{y_1}(\omega, x)$ and $E_{y_2}(\omega, x)$ from equations (12), (14), (22) and (24) to equation (26) to obtain the following dispersion relation of the TM surface waves at the interface between a linear dielectric and a left-handed medium:

$$\frac{k_1}{\varepsilon(\omega)} + \frac{k_2}{\varepsilon_d} = 0 \tag{28}$$

3. The Numerical Results

In this section, we numerically study the dispersion relations (27) and (28) for the TE and TM polarized waves propagating along LHM/RHM interface. In this connection, we show the dependence of wave vector k on the propagation frequency, within the existence frequency band of LHM. For a comparison of dispersion characteristics, we reduce the dispersion relation for LHM/RHM interface to RHM/RHM interface.

To plot the dispersion relations (27) and (28), we first express them as explicit functions of ω and k. For this purpose, we substitute the values of k_1 and k_2, defined in equations (8) and (18), to equations (27) and (28), to obtain the following forms of the dispersion relations:

$$\text{TE Waves : } k = \pm\frac{\omega}{c}\left\{\frac{\mu(\omega)\mu_d(\varepsilon_d\mu(\omega) - \varepsilon(\omega)\mu_d)}{(\mu^2(\omega) - \mu_d^2)}\right\}^{\frac{1}{2}} \tag{29}$$

$$\text{TM Waves : } k = \pm\frac{\omega}{c}\left\{\frac{\varepsilon(\omega)\varepsilon_d(\mu_d\varepsilon(\omega) - \mu(\omega)\varepsilon_d)}{(\varepsilon^2(\omega) - \varepsilon_d^2)}\right\}^{\frac{1}{2}} \tag{30}$$

Equations (29) and (30) represent dispersion relations for TE and TM polarized waves for the RHM/LHM interface. We can obtain the following dispersion relations for the RHM/RHM interface [22] by substituting $\varepsilon(\omega)$, $\mu(\omega)$, ε_d and μ_d by ε_{d_1}, μ_{d_1}, ε_{d_2} and μ_{d_2} respectively in equations (29) and (30):

$$\text{TE Waves : } k = \pm\frac{\omega}{c}\left\{\frac{\mu_{d_1}\mu_{d_2}(\varepsilon_{d_2}\mu_{d_1} - \varepsilon_{d_1}\mu_{d_2})}{(\mu_{d_1}^2 - \mu_{d_2}^2)}\right\}^{\frac{1}{2}} \tag{31}$$

$$\text{TM Waves : } k = \pm\frac{\omega}{c}\left\{\frac{\varepsilon_{d_1}\varepsilon_{d_2}(\mu_{d_2}\varepsilon_{d_1} - \mu_{d_1}\varepsilon_{d_2})}{(\varepsilon_{d_1}^2 - \varepsilon_{d_2}^2)}\right\}^{\frac{1}{2}} \tag{32}$$

3.1. Numerical Analysis of RHM/RHM interface

To obtain the dispersion characteristics of TE and TM surface waves propagating at the interface between two different dielectric media with positive signs of permittivity and permeability, we plot equation (31) and (32) for the numerical values ε_{d_1}=3, μ_{d_1}=4, ε_d=1, μ_{d_2}=2 respectively. Figure 2 shows a plot of propagation frequency ω versus the wave vector k for the surface waves propagating along RHM/RHM interface, for both TE and TM waves and for the whole microwave range of frequency. The graph shows that for both TE and TM polarizations, the surface waves propagate for the whole range of frequency but the propagation is prominent above a frequency $\omega \sim 10^{11} Hz$. It can be seen that the values of wave vector k are very small below $\omega \sim 10^{11} Hz$. This fact is useful for the microwave propagation devices e.g., filters, sensors, frequency selectors etc. Further, the dispersion curves for TE and TM wave modes have a difference in their order of magnitudes as they are separating each other continuously after $\omega \sim 10^{11} Hz$. Therefore, the wave polarizations TE and TM can tune the device according to the desired pair of values for frequency and wave vector.

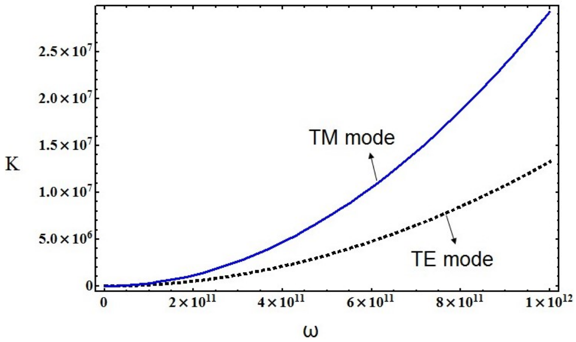

Figure 2: A plot of propagation frequency ω and wave vector k for surface wave propagating at RHM/RHM interface

3.2. Numerical Analysis of RHM/LHM Structure

To plot the dispersion characteristics for a LHM/RHM structure, we first find the frequency range in which the permittivity and permeability have simultaneously negative signs. For this purpose, we plot relative permittivity $\varepsilon(\omega)$ and relative permeability $\mu(\omega)$ given in Equations (1) and (2) against the propagation frequency as shown in Figure 3. For this plot, the numerical values of the physical parameters are given as: F = 0.56, $\omega_r = 4 \times 10^9 Hz$ and $\omega_p = 10 \times 10^9 Hz$ [9, 11]. Figure 3 shows that the frequency range for the simultaneously negative values of permittivity and permeability is from $4GHz$ to $6GHz$.

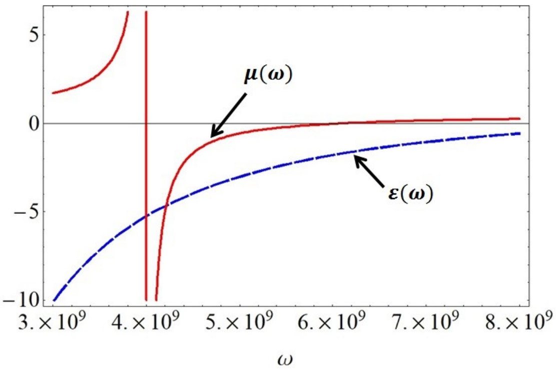

Figure 3: A plot between permittivity and permeability (i.e. $\varepsilon(\omega)$ and $\mu(\omega)$) of left-handed material against the operating frequency ω

To obtain the dispersion characteristics of TE and TM surface waves propagating at the interface between a right handed dielectric medium and a left-handed medium., we use the following numerical values in Equations (29) and (30): F = 0.56, $\omega_r = 4 \times 10^9 Hz$, $\omega_p = 10 \times 10^9 Hz$, $\varepsilon_d = 2$ and $\mu_d = 4$ [9, 11].

Figure 4 shows a plot of surface wave propagation frequency ω versus the wave vector k for the RHM/LHM interface, for both TE and TM waves and for the frequency band of a left-handed medium extended from 4 GHz to 6 GHz. The dispersion curve for TM wave polarization shows continues decrease at higher frequencies which is in contrast to the dispersion characteristics of the TM polarization of RHM/RHM structure. Due to this fact, our proposed RHM/LHM interface can be used to design a low pass filter for the TM surface waves within the existence band of the left-handed material i.e. 4 to 6 GHz.

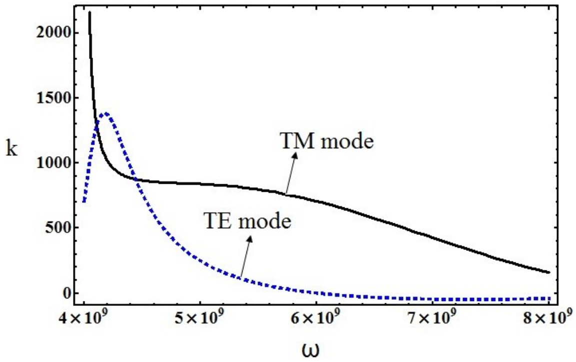

Figure 4: A plot between permittivity and permeability (i.e. $\varepsilon(\omega)$ and $\mu(\omega)$) of left-handed material against the operating frequency ω

Further, it can also be seen that at lower side of the frequency band of the LHM, the dispersion curve for the TE wave polarization shows a sudden increase in amplitude and then it decreases continuously at higher values of frequency. This fact suggests that the proposed interface can be used for the designing of a band pass filter for the TE surface waves (for only lower frequency side of the LHM frequency band).

4. Conclusion

Here, we have presented a way to study the propagation properties and its possible application for the transverse electric and transverse magnetic surface waves at the interface between a linear right-handed and a linear left-handed material. In this connection, the dispersion relations for the TE and TM surface waves have been derived and reduced for the case of linear RHM/RHM interface. These dispersion relations have been plotted numerically for the variation of various physical parameters to discuss the propagation properties of the surface waves.

The dispersion characteristics for the TE and TM waves propagating at the interface of two different right-handed media show that the surface waves propagate for the whole range of frequency but the propagation is prominent above a frequency $\omega \sim 10^{11}$ Hz. It can be seen for very small values of wave vector k below $\omega \sim 10^{11}$ Hz. This fact is useful for the microwave propagation devices e.g. filters, sensors, frequency selectors etc. Further, the dispersion curves for TE and TM wave modes have a difference in their order of magnitudes as they are separating each other continuously after $\omega \sim 10^{11}$ Hz. Therefore, the wave polarizations TE and TM can tune the device according to the desired pair of values for frequency and wave vector.

The dispersion characteristics for the TE and TM waves propagating at the interface of a RHM and a LHM show that the TM wave polarization shows continues decrease at higher frequencies which is in contrast to the dispersion characteristics of the TM polarization of RHM/RHM structure. This fact can be used as a low pass filter within the frequency band of the left-handed material i.e. 4 to 6 GHz. Further, at lower side of the frequency band of the LHM, the dispersion curve for the TE wave polarization shows a sudden increase in amplitude and then it decreases continuously at higher values of frequency. This fact suggests that the proposed interface can be used for the designing of a band pass filter for the TE surface waves.

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